AP® CALCULUS BC

BEGINNING 2016–17 ACADEMIC YEAR

About the Advanced Placement Program® (AP®)

The Advanced Placement Program® enables willing and academically prepared students to pursue college-level studies — with the opportunity to earn college credit, advanced placement, or both — while still in high school. AP Exams are given each year in May. Students who earn a qualifying score on an AP Exam are typically eligible to receive college credit, placement into advanced courses, or both. Every aspect of AP course and exam development is the result of collaboration between AP teachers and college faculty. They work together to develop AP courses and exams, set scoring standards, and score the exams. College faculty review every AP teacher's course syllabus.

AP Calculus Program

AP Calculus AB and AP Calculus BC focus on students' understanding of calculus concepts and provide experience with methods and applications. Although computational competence is an important outcome, the main emphasis is on a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations are important.

Teachers and students should regularly use technology to reinforce relationships among functions, to confirm written work, to implement experimentation, and to assist in interpreting results. Through the use of the unifying themes of calculus (e.g., derivatives, integrals, limits, approximation, and applications and modeling) the courses become cohesive rather than a collection of unrelated topics.

AP Calculus BC Course Overview

AP Calculus BC is roughly equivalent to both first and second semester college calculus courses and extends the content learned in AB to different types of equations and introduces the topic of sequences and series. The AP course covers topics in differential and integral calculus, including concepts and skills of limits, derivatives, definite integrals, the Fundamental Theorem of Calculus, and series. The course teaches students to approach calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and to make connections amongst these representations.

Students learn how to use technology to help solve problems, experiment, interpret results, and support conclusions.

RECOMMENDED PREREQUISITES

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions at the numbers 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, and their multiples.

Use of Graphing Calculators

Professional mathematics organizations have strongly endorsed the use of calculators in mathematics instruction and testing. The use of a graphing calculator in AP Calculus BC is considered an integral part of the course.

AP Calculus BC Course Content

The course is organized around the foundational concepts of calculus:

I. Limits:
Students must have a solid, intuitive understanding of limits and be able to compute one-sided limits, limits at infinity, the limit of a sequence, and infinite limits. They should be able to apply limits to understand the behavior of a function near a point and understand how limits are used to determine continuity.

II. Derivatives:
Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. Students should also be able to solve separable differential equations, understand and be able to apply the Mean Value Theorem, and be familiar with a variety of real-world applications, including related rates, optimization, and growth and decay models.

III. Integrals and the Fundamental Theorem of Calculus:
Students should be familiar with basic techniques of integration, including basic antiderivatives and substitution, and properties of integrals. Students should also understand area, volume, and motion applications of integrals, as well as the use of the definite integral as an accumulation function. It is critical that students understand the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus.

IV. Series:
Students should be familiar with various methods for determining convergence and divergence of a series, Maclaurin series for common functions, general Taylor series representations, radius and interval of convergence, and operations on power series. The technique of using power series to approximate an arbitrary function near a specific value allows for an important connection back to the tangent-line problem.

Mathematical Practices for AP Calculus

The Mathematical Practices for AP Calculus (MPACs) capture important aspects of the work that mathematicians engage in, at the level of competence expected of AP Calculus students. These MPACs are highly interrelated tools that should be used frequently and in diverse contests to support conceptual understanding of calculus.

1. Reasoning with definitions and theorems
2. Connecting concepts
3. Implementing algebraic/computational processes
4. Connecting multiple representations
5. Building notational fluency
6. Communicating

AP Calculus BC Exam Structure

AP CALCULUS BC EXAM: 3 HOURS 15 MINUTES

Assessment Overview

The AP Calculus BC Exam questions measure students' understanding of the concepts of calculus, their ability to apply these concepts, and their ability to make connections among graphical, numerical, analytical, and verbal representations of mathematics. Adequate preparation for the exam also includes a strong foundation in algebra, geometry, trigonometry, and elementary functions, though the course necessarily focuses on differential and integral calculus. Students may not take both the Calculus AB and Calculus BC Exams within the same year. A Calculus AB sub-score is reported based on performance on the portion of the Calculus BC Exam devoted to Calculus AB topics.

The free-response section tests students' ability to solve problems using an extended chain of reasoning. During the second timed portion of the free-response section (Part B), students are permitted to continue work on problems in Part A, but they are not permitted to use a calculator during this time.

Format of Assessment

Section I: Multiple Choice | 45 Questions | 105 minutes | 50% of Exam Score

- **Part A:** 30 questions; 60 minutes (no calculator permitted)
- **Part B:** 15 questions; 45 minutes (graphing calculator permitted)

Section II: Free Response | 6 Questions | 90 minutes | 50% of Exam Score

- **Part A:** 2 problems; 30 minutes (graphing calculator permitted)
- **Part B:** 4 problems; 60 minutes (no calculator permitted)

AP CALCULUS BC SAMPLE EXAM QUESTIONS

Sample Multiple-Choice Question

Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$ is true?

(A) The series converges absolutely.

(B) The series converges conditionally.

(C) The series converges, but neither conditionally nor absolutely.

(D) The series diverges.

AP Calculus BC Exam Structure

Sample Free-Response Question

Free Response: Section II, Part B

No calculator is allowed for problems on this part of the exam.

The function f has derivatives of all orders at $x = 0$, and the Maclaurin series for f is

$$\sum_{n=2}^{\infty} \frac{\ln n}{3^n n^3} x^n.$$

(a) Find $f'(0)$ and $f^{(4)}(0)$.

(b) Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(c) Using the ratio test, determine the interval of convergence of the Maclaurin series for f. Justify your answer.

Educators: apcentral.collegeboard.org/apcalcbc

Students: apstudent.collegeboard.org/apcalcbc

CollegeBoard

00148-013 (Updated November 2015)